Universality of Decoherence

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We consider environment induced decoherence of quantum superpositions to mixtures in the limit in which that process is much faster than any competing one generated by the Hamiltonian H_{sys} of the isolated system. While the golden rule then does not apply we can discard H_{sys}. By allowing for simultaneous couplings to different reservoirs, we reveal decoherence as a universal short-time phenomenon independent of the character of the system as well as the bath and of the basis the superimposed states are taken from. We discuss consequences for the classical behavior of the macroworld and quantum measurement: For the decoherence of superpositions of macroscopically distinct states the system Hamiltonian is always negligible.

Superpositions of quantum states give rise to interference effects which are, however, more and more difficult to observe as the size of the system is increased and the superimposed states are made more distinct. While quantum interferences are ubiquitous in the microworld, none have been seen for macroscopic bodies. A parameter controlling the discernability of interference fringes is the ratio of the de Broglie wavelength λ of a particle to a typical linear dimension Δ of the spatial structure used to construct superpositions of different partial particle waves. When that parameter is of order unity, interference is easily measurable; but upon decreasing λ/Δ, either by letting Δ grow or using ever more massive particles and thus ever smaller λ, wave effects become elusive and eventually escape current detection techniques. As a concrete example, we may think of double-slit experiments (with Δ the slit distance) for which the increasing difficulty in question concerns the resolution of angular apertures of diffraction structures of order λ/Δ.

It is nowadays widely accepted that an even more important reason for the notorious absence of quantum superpositions from the macroscopic world lies in environment imposed decoherence [1,2], and that phenomenon is our concern here. Decoherence is, for microscopic bodies, just a facet of dissipation caused by interactions with many-freedom surroundings. Spontaneous emission of light by an atom is such a dissipative process, with the electromagnetic field acting as a weakly coupled environment. Inasmuch as radiatively coupled states of an atom are only "microscopically" distinct it would appear overly pedantic and even somewhat misleading to attach the fashionable label decoherence to polarization decay while reserving dissipation to the exponential decrease of the population of the excited state. But "larger" systems under the influence of environments do invite such different names for the following reason. If two sufficiently distinct states $|s\rangle, |s'\rangle$ are brought to an initial superposition $|\rangle = c|s\rangle + c'|s'\rangle$, the density operator $\rho(t)$ starts out as the projector $\rho(0) = |\rangle\langle|$ and then, for suitable coupling to the environment (see below), decoheres to the mixture $|c|^2|s\rangle\langle s| + |c'|^2|s'\rangle\langle s'|$, with the weights $|c|^2, |c'|^2$ still as in the initial superposition, on a time scale τ_{dec} while the subsequent relaxation of that mixture towards an eventually stationary $\rho(\infty)$ has a much longer characteristic time τ_{diss}. The time scale ratio $\tau_{\text{dec}}/\tau_{\text{diss}}$ becomes the smaller the more distinct the two component states are. If that distinction can be measured by a length $\Delta \propto |s - s'|$ the time scale ratio is again determined by the ratio of the de Broglie wavelength to Δ, typically as

$$\tau_{\text{dec}}/\tau_{\text{diss}} \sim (\lambda/\Delta)^2 \ . \tag{1}$$

The quadratic rather than just linear dependence of the "acceleration of decoherence against dissipative equilibration of probabilities" on the ratio λ/Δ suggests that decoherence gives rise to classical behavior of the macroworld. Indeed, for macroscopic bodies and macroscopic values of Δ the acceleration factor is typically so small that decoherence appears as instantaneous while dissipation for classically meaningful quantities may not at all be noticeable.

The acceleration factor in question has been studied in recent experiments [3–5]. Arndt et al. in Vienna [3] observed multiple-slit diffraction of the largest objects thus far, C_{60} molecules. No decohering influence of the environment was effective, simply because τ_{diss} and the relevant thermal de Broglie length were sufficiently large. Experiments at the ENS in Paris [4] involved superpositions of coherent states of a microwave cavity mode. Even though the cavity was of high quality ($\tau_{\text{diss}} = 160\mu\text{s}$) the acceleration factor was controlled between, roughly, 1 and 10. Finally, a NIST group [5] worked with superpositions of translational-motion states of single $^9\text{Be}^+$ atoms in Paul traps. Here, the acceleration factor was steered through the range $1 \ldots 25$ and the environment engineered so as to even vary τ_{diss}.

On the theoretical side the experiments mentioned are well understood. In all cases the decoherence time is not smaller than the time scale of dissipation by many orders of magnitude; it is in fact still larger than the characteristic times of the free motion of the system in the absence of

any environment. In that limit, both dissipation and its companion decoherence can be treated by Fermi's golden rule or fancied-up variants thereof like master equations. The golden rule involves a certain long-time limit: It cannot yield time independent transition rates (or Markovian master equations) before the time elapsed since the preparation of the initial state exceeds the basic periods τ_{sys} of the isolated system. Indeed, inasmuch as it explicitly requires energy conservation for the exchange of free-bath and free-system quanta, it presupposes such large times for resonance to become effective.

The golden rule cannot be trusted when it predicts a decoherence time τ_{dec} smaller than τ_{sys}. It therefore does not help to explain why the macroworld behaves classically. When nevertheless holding decoherence responsible here we mostly rely on an exactly solvable model, a harmonic oscillator harmonically coupled to a bath itself consisting of harmonic oscillators [6]. The acceleration factor $(\lambda/\Delta)^2$ is there found in effect already for arbitrarily small times. Invaluable as the oscillator model is for rigorously revealing decoherence, we would like to prove a purportedly universal phenomenon to emerge universally, rather than only for a very special model.

The clue to progress lies in the fact that the Hamiltonian $H = H_{\text{sys}} + H_{\text{bath}} + H_{\text{int}}$ of the embedding of a system in an environment (alias bath) can be simplified for times much smaller than the characteristic times τ_{sys} of H_{sys}: We can altogether neglect any motion the isolated system would perform, i. e. discard H_{sys}. For the structure of the interaction Hamiltonian we do not have much of a choice. Introducing a coupling agent each for the system, S, and the bath, B, we may write

$$H_{\text{int}} = SB. \tag{2}$$

It follows that the coupling agent S becomes conserved and only plays the role of a fixed parameter. Introducing eigenvectors and eigenvalues of S as $S|s\rangle = s|s\rangle$ we consider the matrix element $\langle s|W|s'\rangle$ of the joint density operator of our compound which still is a density operator for the bath. We shall eventually be interested only in the reduced density matrix $\langle s|\rho|s'\rangle = \text{Tr}_{\text{bath}}\langle s|W|s'\rangle$ and intend to rigorously reveal decoherence as a universal short-time phenomenon by showing

$$\langle s|\rho(t)|s'\rangle = e^{-(s-s')^2 f(t) + i(s^2 - s'^2)\,\varphi(t)}\,\langle s|\rho(0)|s'\rangle\,, \tag{3}$$

with functions $f(t) \geq 0$ and $\varphi(t)$ to be determined.

For the next step we momentarily model the bath as a collection of oscillators and specify

$$H_{\text{bath}} = \sum_{i=1}^{N}\Big(\frac{1}{2m}\hat{p}_i^2 + \frac{1}{2}m\omega_i^2\hat{q}_i^2\Big)\,,\ H_{\text{int}} = S\sum_{i=1}^{N} g_i\hat{q}_i\,. \tag{4}$$

To write the Liouville-von Neumann equation for the joint density operator W it is convenient to stick to the S-representation and employ the Wigner function with respect to the bath oscillators. Denoting by $W(s, s', p, q, t)$

that hybrid representative we get the reduced density matrix by integrating over the $(2N)$-dimensional phase space of the bath, $\langle s|\rho(t)|s'\rangle = \int d^N p d^N q W(s, s', p, q, t)$. The evolution equation $\dot{W} = LW$ has the generator

$$L = \sum_i \Big[\frac{\partial}{\partial p_i}m\omega_i^2 q_i - \frac{\partial}{\partial q_i}\frac{p_i}{m} - g_i\Big[\frac{i}{\hbar}(s - s')q_i - \frac{s + s'}{2}\frac{\partial}{\partial p_i}\Big]\Big].$$

The purely parametric role of the eigenvalues s, s' of the system coupling agent is manifest in the generator L. We could proceed to solving the foregoing first-order differential equation for $W(s, s', p, q, t)$. It is more convenient to directly go for the time evolution of the reduced density matrix. To that end we may assume initial statistical independence of system and bath, $W(s, s', p, q, 0) = \langle s|\rho(0)|s'\rangle \times W_{\text{bath}}(p, q, 0)$. Without loss of generality we momentarily assume the initial bath distribution sharp, $W_{\text{bath}}(p, q, 0) = \prod_i \delta(p_i - p_{i0})\delta(q_i - q_{i0})$, since we may later average with whatever weight we please. A reduced time evolution operator can be introduced as $U(t) = \int d^N p d^N q \, e^{Lt} W_{\text{bath}}(p, q, 0)$. We readily check

$$\dot{U} = -\frac{i}{\hbar}(s - s')\sum_i g_i \int d^N p d^N q \, q_i \, e^{Lt} W_{\text{bath}}(p, q, 0) \tag{5}$$

and then, using the commutator $[\frac{\partial}{\partial q_i}, q_j] = \delta_{ij}$, shift the factor q_i in the integrand to the right of the exponential e^{Lt}, to get a reduced generator $l(t) = \dot{U}(t)U(t)^{-1}$ as

$$l(t) = \frac{i}{\hbar}\sum_i \Big[(s^2 - s'^2)\frac{g_i^2}{2m\omega_i^2}(1 - \cos\omega_i t)$$
$$-(s - s')g_i\big(q_{i0}\cos\omega_i t + \frac{p_{i0}}{m\omega_i}\sin\omega_i t\big)\Big]. \tag{6}$$

Due to the parametric role of the eigenvalues s, s' we here do not confront a differential operator and get the density matrix as $\langle s|\rho(t)|s'\rangle = \exp\{\int_0^t dt' l(t')\}\,\langle s|\rho(0)|s'\rangle$; this contains the initial coordinates q_{i0} and momenta p_{i0} in the exponent. Now we invoke a thermal bath and average as $\overline{e^{iaq_i}} = e^{-a^2\hbar/4m\omega_i \tanh(\beta\hbar\omega_i/2)}$ and $\overline{e^{ibp_i}} = e^{-b^2 m\hbar\omega_i/4 \tanh(\beta\hbar\omega_i/2)}$. The result (3) is so reached with

$$f(t) = \sum_i \frac{g_i^2(1 + 2\overline{n}_i)}{2m\hbar\omega_i^3}\big(1 - \cos\omega_i t\big) = \text{Re}\frac{1}{\hbar^2}\int_0^t ds\, s\, C(t - s)\,,$$

$$\varphi(t) = \sum_i \frac{g_i^2}{2m\hbar\omega_i^2}\big(t - \frac{\sin\omega_i t}{\omega_i}\big) = \text{Im}\frac{1}{\hbar^2}\int_0^t ds\, s\, C(t - s)\,, \tag{7}$$

where $\overline{n}_i = (e^{\beta\hbar\omega_i} - 1)^{-1}$ is the thermal number of quanta and $C(t) = \langle B(t)B(0)\rangle$ the thermal autocorrelation function of the bath coupling agent. The function $f(t)$, which determines the decoherence as thermally enhanced by the factor $1 + 2\overline{n}_i$, begins quadratically in t; for larger times it approaches $f(t) \to \gamma t$ with $\gamma = \text{Re}\,\hbar^{-2}\int_0^\infty dt C(t)$, provided $C(t)$ falls off faster than t^{-2}. The proportionality

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of $f(t)$ to $\hbar^{-1}$ signals a quantum scale λ^2 of reference for $(s - s')^2$. The phase $\varphi(t)$ begins $\propto t^3$ and is temperature independent. We should appreciate the dramatic difference of the decoherence function $f(t)$ from its golden-rule counterpart. If the system were itself an harmonic oscillator with frequency Ω, mass M, and displacement S the golden-rule would yield $f_{\mathrm{GR}}(t) = \gamma^{\mathrm{GR}} t$ with $\gamma^{\mathrm{GR}} = (1 + 2\overline{n}(\Omega))(2\hbar)^{-2}\mathrm{Re}\int_0^\infty dt e^{\mathrm{i}\Omega t}\langle[B(t), B(0)]\rangle$. Most importantly, our decoherence function in (3) describes accelerated decoherence for whatever system with whatever coupling agent S, provided only decoherence is fast in the sense $\tau_{\mathrm{dec}} \ll \tau_{\mathrm{sys}}$ which will always be the case for sufficiently distinct s and s'.

We have not gone more than half way towards our goal yet. The decoherence shown by the foregoing reasoning is a privilege of superpositions of eigenstates of the system coupling agent; superpositions of eigenstates of other system observables not commuting with S do not lose their relative quantum phases any faster than probabilities change. But on the other hand no privileged representations are known in the macroworld; no quantum coherence has ever been seen between any pair of macroscopically distinct states.

For fast decoherence of macroscopic superpositions to take place without distinction of special observables or states, a variety of environmental influences would have to be at work. Rather than privileging a single observable S as the one and only coupling agent to contact one and only one reservoir it seems necessarry to account for several non-commuting system observables as coupling agents toward several reservoirs.

To model the situation just sketched we accompany the single agent S by a canonically conjugate partner R with $[R, S] = \hbar/\mathrm{i}$. More general models would involve a larger set of non-commuting system observables but would not lead to conclusions qualitatively different from the ones to be discussed here. Still interested in times smaller than τ_{sys} we need not worry about a system Hamiltonian H_{sys} and generalize (4) to the double-bath Hamiltonian

$$H_{\mathrm{bath}} = \sum_i \left[\frac{1}{2m}\hat{p}_i^2 + \frac{1}{2}m\omega_i^2\hat{q}_i^2\right] + \sum_i \left[\frac{1}{2M}\hat{P}_i^2 + \frac{1}{2}M\Omega_i^2\hat{Q}_i^2\right],$$

$$H_{\mathrm{int}} = S\sum_i g_i\hat{q}_i + R\sum_i G_i\hat{Q}_i \,.$$

Neither S nor its conjugate partner R now play a merely parametric role. Even though the neglect of H_{sys} still forbids the appearance of any nonlinearity and guarantees explicit tractability with the same strategy as above, the resulting expressions are so unwieldy to not warrant full display. The generator L gets an addition differing from the previous term only by $\{p_i, q_i, \omega_i, m\} \to \{P_i, Q_i, \Omega_i, M\}$ and $s \to \frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial s}$, $s' \to \mathrm{i}\hbar\frac{\partial}{\partial s'}$, the latter two replacements familiar for a momentum in the position representation. The corresponding addition arises for the time rate of change of the reduced evolution op-

erator $\dot{U}(t)$ in (5) but from that point on an avalanche of complications is set loose: When scrutinizing $e^{-Lt}q_i e^{Lt}$ and $e^{-Lt}Q_i e^{Lt}$ we find all variables $\{q_i, p_i, Q_i, P_i, s, \frac{\partial}{\partial s}\}$ coupled. The analogue of the reduced generator (6) is bulky and, worse, contains non-commuting pieces and thus gives rise to a yet bulkier time ordered exponential $(\exp\int_0^t dt' l(t'))_+$. Fortunately, to within corrections of order t^3 we can drop all non-commuting terms in the new $l(t)$ and arrive at the reduced density matrix

$$\langle s|\rho(t)|s'\rangle = e^{-(s-s')^2 f(t)}\, e^{\hbar^2(\frac{\partial}{\partial s} + \frac{\partial}{\partial s'})^2 F(t)}\, \langle s|\rho(0)|s'\rangle \quad (8)$$

where $f(t)$ is given by (7) and $F(t)$ likewise except for $(m, \omega_i, g_i) \to (M, \Omega_i, G_i)$. To make peace with the absence of a phase factor from (8) it is well to recall from (7) that $\varphi(t) \propto t^3$ for small times. Upon Fourier transforming to the momentum representation we see that the second exponential in (8) entails accelerated decay for coherences with respect to eigenstates of the momentum R just as the first factor does for the eigenstates of S.

As expected, by introducing different reservoirs coupling to non-commuting observables we break the privilege of a single representation. Non-commuting agents contacting different reservoirs occur for a body probing an electric field through its charge or electric dipole moment and a magnetic field with its magnetic moment. That simple example indicates that for a macroscopic body the exclusive action of a single reservoir may be as unrealistic a fiction as complete isolation.

Our discussion is of relevance for the quantum measurement problem. It had long been considered a puzzle how a microscopic object prepared in a superposition of, say, two eigenstates of an observable to be measured can, through unitary evolution of its composition with a macroscopic pointer, cause that pointer to reach one of two distinct positions in each run of the measurement, with those positions uniquely related to the two eigenvalues and repeated runs building up probabilities equal to the weights in the original superposition. Enigmatic was not the entanglement of microobject and pointer into a superposition associating each eigenstate of the measured microobservable with a unique pointer state (a superposition of the type often called a Schrödinger cat state); such entanglement is accessible through unitary evolution, as was already explained by von Neumann [7]. The puzzle rather was the collapse of that superposition to the mixture with unchanged probabilities. The current understanding is [1,2], that a many-freedom surrounding decoheres the superposition. Zurek has pointed out that decoherence of different pointer displacements is most easily understood if the pointer displacement is taken as the pointer's coupling agent towards a reservoir. The prize to be paid is the distinction of a "pointer basis" at least for times up to τ_{sys}. We now see that no such prize is due when the pointer displacement is not the

only coupling agent but just one of several, each towards a different reservoir.

Can decoherence be reversed? Like any other dissipative phenomenon in (subdynamics of) unitary evolutions, decoherence should not be considered absolutely irreversible. Seemingly spontaneous revivals of coherences out of an apparent mixture could arise for a macroscopic system, in the unitary motion of its composition with an environment, given a suitable initial state of the composition. Likewise, the time reversal of decoherence is not in conflict with unitarity of the composite dynamics and again requires "no more than" suitable initial conditions. As regards ordinary damping, such reversals were demonstrated in the historic spin echo experiments [8]; but decoherence is just ordinary damping, starting from an extra-ordinary initial state.

While decoherence can explain why quantum superpositions are alien to the macroworld, it does by no means imply that quantum behavior never reaches out to the macroscopic. For instance, once the initial state of the object-pointer compound of a measurement process has decohered, the pointer has vanishingly small probability to jump between the various positions it could have gone to by the previous interaction with the micro-object. The pointer will rather keep moving classically, up to tiny fluctuations. What is left from the entangled state of the micro-object and the pointer are finite probabilities for various pointer displacements and these are due to the extremely nonclassical nature of the initial state.

Our conclusions about the simultaneous action of several reservoirs will not be invalidated when $f(t)$ and $F(t)$ begin to deviate from their $\mathcal{O}(t^2)$ approximants. If we do not attempt to write out corrections it is for lack of space and because the corrections are system specific. The decoherence manifest in the $\mathcal{O}(t^2)$ terms is universal, however, even beyond the harmonic-oscillator baths. To see this we may glance back at our procedure from the reduced generator (6) to the density matrix (3,7) and its generalization to two baths. The $\mathcal{O}(t^2)$ behavior of the exponentials in the density matrices (3,8) could have been gotten most easily by restricting ourselves to the initial form $l(0)$ of the reduced generator: Indeed, the time integral $\int_0^t dt' l(t') = l(0)t + \dots$ and the subsequent thermal average of its exponential precisely gives the $\mathcal{O}(t^2)$ terms in the exponents in the final density matrices (3,8); but confining ourselves to $l(0)$ means neglecting even the free Hamiltonian H_bath of the reservoirs and would thus not yield anything different for quite different bath models like, say, collections of anharmonic oscillators. The possibility of ascertaining coherence loss from short-time expansions was also recognized in Ref. [9]. At any rate, rapid decoherence of macroscopic superpositions is obviously a universal phenomenon.

We owe a final remark to present efforts towards realizing quantum computing. A quantum computer would encorporate lots of quantum rather than classical two-state elements and would therefore be a mesoscopic or macroscopic complex. The whole complex would seem prone to accelerated decoherence. No computation relying on coherences seems possible during time spans exceeding τ_dec. Hope must therefore be set on error correction codes or elements which are sufficiently decoupled from detrimental environments. Symmetry may be of help [10,11]. For a simple example, imagine a coupling agent of the form $S = \Sigma^2$ such that Σ is itself an observable which has a pair of eigenvalues $\pm\sigma$. A reservoir contacted through the agent S could not decohere a superposition within the subspace of degeneracy, $c_+|\sigma\rangle + c_-|-\sigma\rangle$, even if the eigenvalues $\pm\sigma$ were macroscopically distinct.

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